

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



General Certificate of Education  
Advanced Level Examination  
June 2015

# Mathematics

# MPC3

## Unit Pure Core 3

Friday 5 June 2015 9.00 am to 10.30 am

**For this paper you must have:**

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

**Time allowed**

- 1 hour 30 minutes

- Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
  - Fill in the boxes at the top of this page.
  - Answer **all** questions.
  - Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
  - You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
  - Do not write outside the box around each page.
  - Show all necessary working; otherwise marks for method may be lost.
  - Do all rough work in this book. Cross through any work that you do not want to be marked.

- Information**
- The marks for questions are shown in brackets.
  - The maximum mark for this paper is 75.

- Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
  - You do not necessarily need to use all the space provided.



J U N 1 5 M P C 3 0 1

Answer **all** questions.

Answer each question in the space provided for that question.

- 1 (a)** Use the mid-ordinate rule with four strips to find an estimate for  $\int_{1.5}^{5.5} e^{2-x} \ln(3x-2) dx$ , giving your answer to three decimal places.

**[4 marks]**

- (b)** Find the exact value of the gradient of the curve  $y = e^{2-x} \ln(3x-2)$  at the point on the curve where  $x = 2$ .

**[4 marks]**

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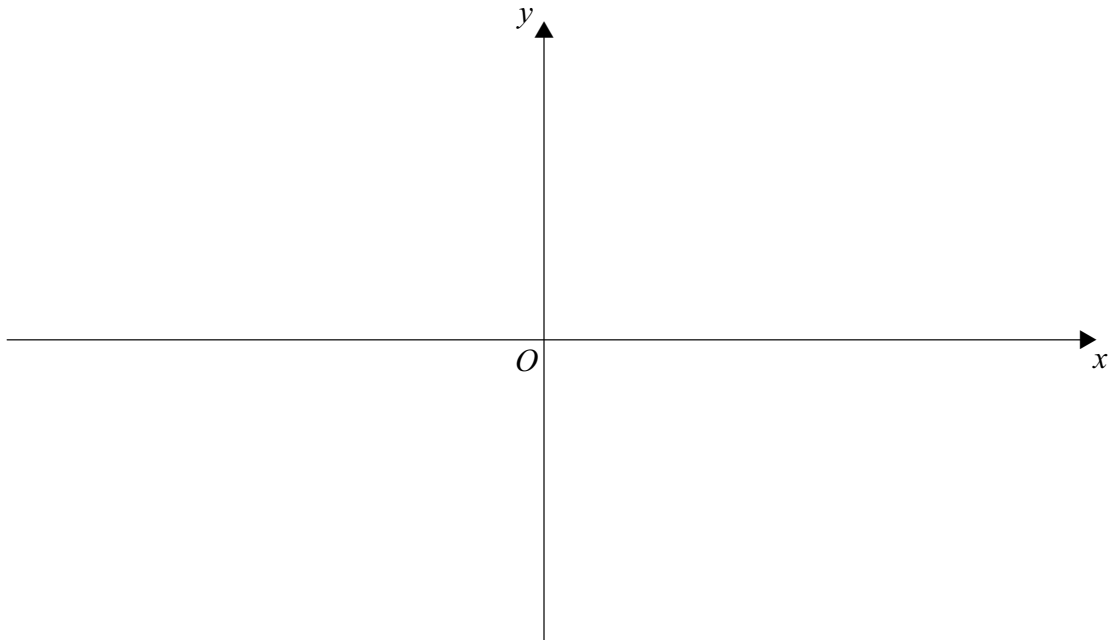


- 2 (a)** Sketch, on the axes below, the curve with equation  $y = 4 - |2x + 1|$ , indicating the coordinates where the curve crosses the axes. **[4 marks]**
- (b)** Solve the equation  $x = 4 - |2x + 1|$ . **[3 marks]**
- (c)** Solve the inequality  $x < 4 - |2x + 1|$ . **[2 marks]**
- (d)** Describe a sequence of two geometrical transformations that maps the graph of  $y = |2x + 1|$  onto the graph of  $y = 4 - |2x + 1|$ . **[4 marks]**

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**(a)**



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**3 (a)** It is given that the curves with equations  $y = 6 \ln x$  and  $y = 8x - x^2 - 3$  intersect at a single point where  $x = \alpha$ .

(i) Show that  $\alpha$  lies between 5 and 6. **[2 marks]**

(ii) Show that the equation  $x = 4 + \sqrt{13 - 6 \ln x}$  can be rearranged into the form

$$6 \ln x + x^2 - 8x + 3 = 0$$
**[3 marks]**

(iii) Use the iterative formula

$$x_{n+1} = 4 + \sqrt{13 - 6 \ln x_n}$$

with  $x_1 = 5$  to find the values of  $x_2$  and  $x_3$ , giving your answers to three decimal places. **[2 marks]**

**(b)** A curve has equation  $y = f(x)$  where  $f(x) = 6 \ln x + x^2 - 8x + 3$ .

(i) Find the exact values of the coordinates of the stationary points of the curve. **[5 marks]**

(ii) Hence, or otherwise, find the exact values of the coordinates of the stationary points of the curve with equation

$$y = 2f(x - 4)$$
**[2 marks]**

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4 The functions  $f$  and  $g$  are defined by

$$f(x) = 5 - e^{3x}, \text{ for all real values of } x$$

$$g(x) = \frac{1}{2x - 3}, \text{ for } x \neq 1.5$$

(a) Find the range of  $f$ . **[2 marks]**

(b) The inverse of  $f$  is  $f^{-1}$ .

(i) Find  $f^{-1}(x)$ . **[3 marks]**

(ii) Solve the equation  $f^{-1}(x) = 0$ . **[1 mark]**

(c) Find an expression for  $gg(x)$ , giving your answer in the form  $\frac{ax + b}{cx + d}$ , where  $a, b, c$  and  $d$  are integers. **[3 marks]**

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**5 (a)** By writing  $\tan x$  as  $\frac{\sin x}{\cos x}$ , use the quotient rule to show that  $\frac{d}{dx}(\tan x) = \sec^2 x$ .

**[2 marks]**

**(b)** Use integration by parts to find  $\int x \sec^2 x \, dx$ .

**[4 marks]**

**(c)** The region bounded by the curve  $y = (5\sqrt{x}) \sec x$ , the  $x$ -axis from 0 to 1 and the line  $x = 1$  is rotated through  $2\pi$  radians about the  $x$ -axis to form a solid.

Find the value of the volume of the solid generated, giving your answer to two significant figures.

**[3 marks]**

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**6 (a)** Sketch, on the axes below, the curve with equation  $y = \sin^{-1}(3x)$ , where  $y$  is in radians.

State the exact values of the coordinates of the end points of the graph.

**[3 marks]**

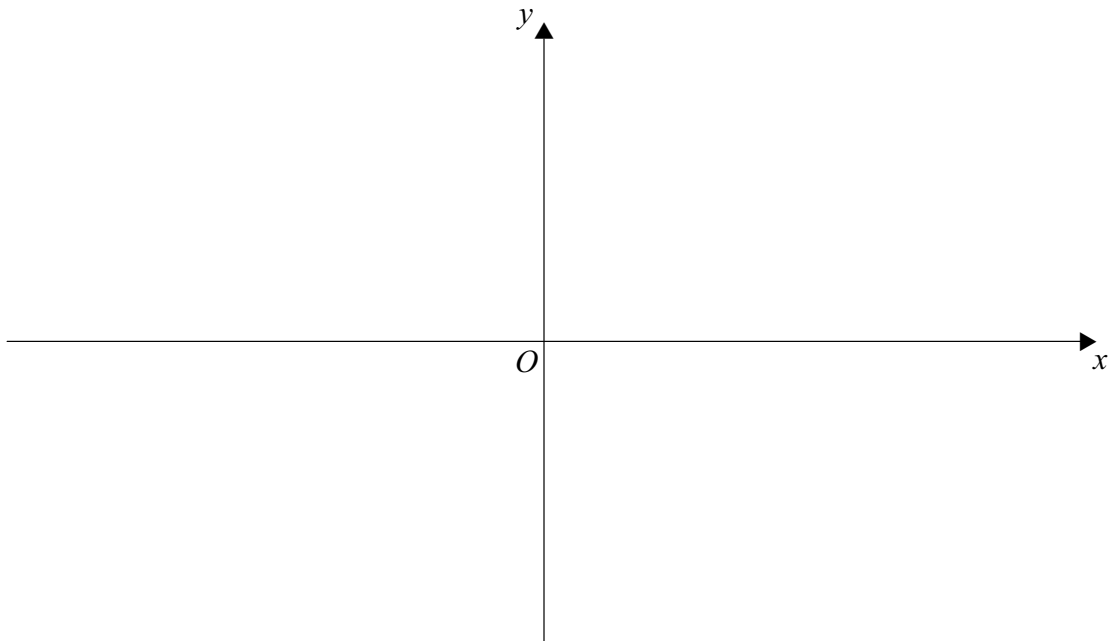
**(b)** Given that  $x = \frac{1}{3}\sin y$ , write down  $\frac{dx}{dy}$  and hence find  $\frac{dy}{dx}$  in terms of  $y$ .

**[2 marks]**

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**(a)**



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- 7 Use the substitution  $u = 6 - x^2$  to find the value of  $\int_1^2 \frac{x^3}{\sqrt{6-x^2}} dx$ , giving your answer in the form  $p\sqrt{5} + q\sqrt{2}$ , where  $p$  and  $q$  are rational numbers.

**[7 marks]**QUESTION  
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- 8 (a) Show that the equation  $4 \operatorname{cosec}^2 \theta - \cot^2 \theta = k$ , where  $k \neq 4$ , can be written in the form

$$\sec^2 \theta = \frac{k-1}{k-4}$$

[5 marks]

- (b) Hence, or otherwise, solve the equation

$$4 \operatorname{cosec}^2(2x + 75^\circ) - \cot^2(2x + 75^\circ) = 5$$

giving all values of  $x$  in the interval  $0^\circ < x < 180^\circ$ .

[5 marks]

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**END OF QUESTIONS**

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